The Impact of Assuming Flatness in the Determination of Neutrino Properties from Cosmological Data

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Cosmological data have provided new constraints on the number of neutrino species and the neutrino mass. However these constraints depend on assumptions related to the underlying cosmology. Since a correlation is expected between the number of effective neutrinos N_{eff}, the neutrino mass $\sum m_\nu$, and the curvature of the universe Ω_k, it is useful to investigate the current constraints in the framework of a non-flat universe. In this paper we update the constraints on neutrino parameters by making use of the latest cosmic microwave background (CMB) data from the ACT and SPT experiments and consider the possibility of a universe with non-zero curvature. We first place new constraints on N_{eff} and Ω_k, with $N_{eff} = 4.03 \pm 0.45$ and $10^3 \, \Omega_k = -4.46 \pm 5.24$. Thus, even when Ω_k is allowed to vary, $N_{eff} = 3$ is still disfavored with 95% confidence. We then investigate the correlation between neutrino mass and curvature that shifts the 95% upper limit of $\sum m_\nu < 0.45$ eV to $\sum m_\nu < 0.95$ eV. Thus, the impact of assuming flatness in neutrino cosmology is significant and an essential consideration with future experiments.

I. INTRODUCTION

Throughout the previous decades experimental cosmology has benefited from accurate measurements of the cosmic microwave background (CMB). The data have determined constraints on several cosmological parameters to remarkable accuracy and the ability to constrain new physics with the CMB continues to improve. Future CMB experiments might even be able to measure B-mode polarization and distinguish between neutrino hierarchy models. However, when constraining new parameters one must be careful when constraints depend on assumptions about the underlying cosmology. For example, a correlation between the neutrino properties and the curvature of the universe is clearly expected since, a higher number of neutrino species or large mass would introduce pre-recombination effects, shifting the positions of the peaks in the angular CMB spectrum (cf. [1, 2]).

Here we present an update on the constraints of the number of neutrino species N_{eff} and the sum of neutrino masses Σm_ν in the framework of non-flat universes with $\Omega_k \neq 0$ combining the Wilkinson Microwave Anisotropy Probe (WMAP) 7-year [3], South Pole Telescope (SPT) [4] and Atacama Cosmology Telescope (ACT) [5] datasets.

The paper is organized as follows. In Section II we give theoretical arguments for why the N_{eff} and Ω_k parameters should be correlated. In Section III we discuss our method of constraining the parameters N_{eff}, Σm_ν, and Ω_k. We present the results of the analysis in Section IV. Finally, in Section V we conclude and discuss the implications of assuming flatness in neutrino cosmology.

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II. THE EFFECTIVE NEUTRINO NUMBER

The effective neutrino number N_{eff} is defined as the contribution of neutrinos to the relativistic degrees of freedom g_*. In a standard physics scenario the particles contributing to the total value of $g_* \simeq 10.75$ are electrons, three neutrinos (and their antiparticles), and photons. Any extra relativistic degrees of freedom can be parameterized in terms of an excess with respect to the standard effective neutrino number $N_{eff} = 3$ (which more precisely is $\simeq 3.046$ after accounting for QED corrections and non-instantaneous decoupling of neutrinos) [6, 7]. The neutrino energy density is:

$$\rho_\nu = N_{eff} \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \rho_\gamma \,, \tag{1}$$

where ρ_γ is the energy density of photons. A first effect of N_{eff} is related to the primordial helium abundance Y_P. Changing N_{eff} affects the freeze-out temperature T_{freeze} during BBN and therefore the final neutron to proton ratio n_n/n_p [8]. Larger N_{eff} means earlier freeze-out, larger n_n/n_p, and larger Y_p.

The effect of N_{eff} on cosmological observables (e.g. CMB anisotropy power spectrum and galaxy power spectrum) is emphasized by the epoch of matter-radiation equality a_{eq}. In particular, for what concern the CMB, an increase in a_{eq} changes the extent of the early Integrated Sachs-Wolfe effect. The relation between a_{eq} and N_{eff} is given by equating energy densities:

$$\rho_{rad} = \rho_m \iff a_{eq} = \frac{1 + 0.227 N_{eff}}{40484 \, \Omega_m \, h^2} \,. \tag{2}$$

This shows a linear relationship for $a_{eq}(N_{eff})$, which

transfers to the baryon to photon ratio at equality [9]:

$$R_{eq} = \frac{3\rho_b}{4\rho_\gamma}\Big|_{a_{eq}},$$
$$= 30496\,\Omega_b\,h^2\,a\,\Big|_{a_{eq}},$$
$$= \frac{1 + 0.227 N_{eff}}{1.3276}\frac{\Omega_b}{\Omega_m}. \qquad (3)$$

The presence of baryons in the relativistic cosmic fluid slows down the sound speed according to the definition,

$$c_s \equiv 1/\sqrt{3(1+R)}, \qquad (4)$$

and so this quantity is also affected at equality by the effective neutrino number. This reflects in the size of sound horizon at a generic time τ [9]:

$$r_s \equiv \int_0^\tau d\tau'\, c_s(\tau'),$$
$$= \int_0^a \frac{da}{a^2 H}\, c_s(a),$$
$$\approx \frac{2}{3k_{eq}}\sqrt{\frac{6}{R_{eq}}}\ln\left\{\frac{\sqrt{1+R}+\sqrt{R+R_{eq}}}{1+\sqrt{R_{eq}}}\right\},$$
$$= \frac{6.612\times 10^{-3}}{H_0\sqrt{\Omega_m\Omega_b h^2}}\ln\left\{\frac{\sqrt{1+R}+\sqrt{R+R_{eq}}}{1+\sqrt{R_{eq}}}\right\}. \qquad (5)$$

The last equations come from assuming the Universe is matter dominated during recombination. As can be seen, the sound horizon depends on N_{eff} through R_{eq}.
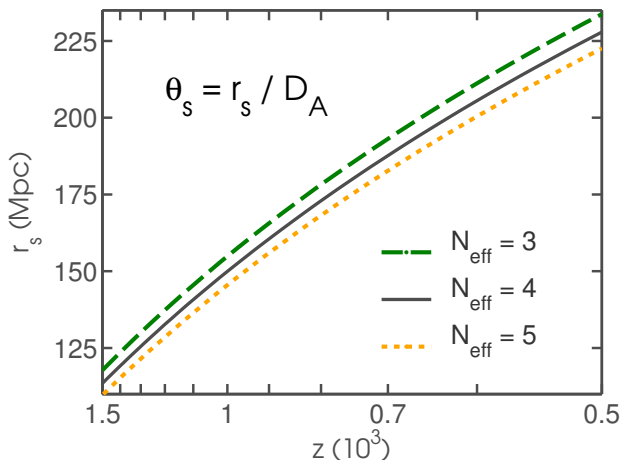

FIG. 1. A demonstration of how the sound horizon r_s changes with the effective neutrino number N_{eff} under the matter dominated approximation given by Eq. 5.

In recent papers (see e.g. [10–12]) it has been found that the number of neutrinos is greater than the standard model value at more than 2σ. The presence of additional neutrinos can be described by a (3+1) or (3+2) model with three active neutrinos and one or two sterile neutrinos [13, 14]. More exotic solutions may include

arguments supporting modified dark energy models [15–17]. In this work we explore possible overlooked parameter degeneracies that could still favor the standard model without the introduction of new physics.

In [10] the authors provide qualitative arguments for how changing the number of allowed neutrinos affects the observed values of parameters. One example which we use is the relative dependence of distance measurements on the Hubble constant. In fact, the sound horizon at recombination scales as $r_s \propto 1/H$ while the distance a photon typically diffuses prior to its last scattering goes as $r_d \propto 1/\sqrt{H}$. This is significant because the response of the radiation relative to matter determines the degree of damping prior to recombination. In other words, with $\theta_s = r_s/D_A$ fixed by observation, the angular diameter distance, D_A, must also decrease as $1/H$ which is more rapid than r_d. Thus, the damping increases according to $\theta_d = r_d/D_A \propto \sqrt{H}$ [10]. If these distances vary according to H then they also vary according to any parameter correlated with H. In an open universe with nonzero curvature the effective neutrino number is slightly reduced. The theory confirms this because as stated above θ_s is constrained by observation which means if N_{eff} is reduced and $\Omega_k > 0$ then r_s and D_A both increase (see Fig. 1). However, if the parameter space favors a closed universe then there will appear to be a higher number of effective neutrinos. This is one of the primary reasons for expecting correlation between N_{eff} and Ω_k.

We conclude this section by stating our purpose further constrain the neutrino mass and investigate the effect of curvature on this parameter. Finally, we acknowledge that there are many papers on the subject of constraining neutrino parameters. A non-exhaustive list of additional references includes Refs. [18–31].

III. ANALYSIS METHOD

In order to fit cosmological models to data we use a modified version of the publicly available CosmoMC software package [32]. This uses a Monte Carlo Markov Chain analysis on calculations of the lensed CMB power spectrum made with the CAMB package. Our analysis combines the following CMB anisotropy datasets: WMAP 7-year [3], SPT [4], and ACT [5]. Including BAO+H_0 simply means we are using the baryon acoustic oscillation (BAO) data of Percival *et al.* [33] and impose a prior on the Hubble parameter based on the last Hubble Space Telescope observations [34]. We integrate spectral data out to $\ell_{\max} = 3000$. We sample from the following parameters: the baryon $\Omega_b h^2$, cold dark matter $\Omega_c h^2$, and dark Ω_Λ energy densities, the scalar spectral index n_s, the optical depth to reionization τ, the Hubble parameter H_0, and the amplitude of SZ spectrum A_{SZ}. We also consider the effective neutrino number N_{eff}, spatial curvature Ω_k, and the sum of neutrino masses $\sum m_\nu$.

Finally, we make decisions specific to the high multipole mode data. We consider purely adiabatic initial con-

Parameter	WMAP7+BAO+H_0	WMAP7+SPT	WMAP7+ACT	WMAP7+SPT+ACT
$100\Omega_b h^2$	2.249 ± 0.054	2.256 ± 0.041	2.235 ± 0.047	2.258 ± 0.040
$\Omega_c h^2$	0.135 ± 0.016	0.130 ± 0.0094	0.137 ± 0.012	0.129 ± 0.0091
Ω_Λ	0.721 ± 0.018	0.722 ± 0.015	0.714 ± 0.018	0.722 ± 0.015
n_s	0.979 ± 0.015	0.9808 ± 0.0122	0.982 ± 0.013	0.9803 ± 0.0121
τ	0.086 ± 0.014	0.085 ± 0.014	0.086 ± 0.014	0.086 ± 0.014
H_0 (km/s/Mpc)	75.1 ± 3.4	74.0 ± 2.0	74.6 ± 2.15	73.9 ± 1.92
N_{eff}	4.34 ± 0.88	3.91 ± 0.43	4.30 ± 0.58	3.89 ± 0.41

TABLE I. Summary of matching results from WMAP 7-year [3], SPT [4], and ACT [5]. Note that the analyses are modeled by the choice to reproduce SPT results, which produces a smaller value for N_{eff} than expected for ACT data. All datasets include BAO and H_0 for improved parameter constraints. The quoted errors are given at the 68% confidence levels (CL).

ditions. When the background data are taken to small enough scales the spectra from infrared source emission must be taken into account. The IR spectra is dominated by Poisson power partially from source emission clustering at the smallest scales. Thus, a model for such effects must be subtracted out from the CMB power spectra. The resulting adaptations are representative of the considerations made during the process of checking the code for consistency with established results. It is also important to include a Big Bang nucleosynthesis (BBN) consistency check during the sampling in order to provide analysis consistent with helium abundance measurements, as proposed in Refs. [35, 36]. We remark that the ACT collaboration did not include the same BBN consistency condition used by SPT and our analysis. This explains why we find a slightly better constraint on N_{eff} than Ref. [5].

Additionally, we also constrain the sum of the neutrino masses Σm_ν. To do this, we use a top hat prior on the fractional contribution of neutrinos to the total mass density, $f_\nu \equiv \Omega_\nu/\Omega_m \in [0, 0.5]$. Then we extract $\sum m_\nu$ from f_ν through the standard relation,

$$\sum m_\nu = 94\Omega_\nu h^2 \text{ eV} = 94h^2\Omega_m f_\nu \text{ eV}, \qquad (6)$$

where $\Omega_\nu \equiv \rho_\nu^0/\rho_{cr}$ is the neutrino contribution to the energy density.

IV. RESULTS

Under the flat Universe scenario the constraint improves to $N_{eff} = 3.89 \pm 0.41$ at the 68% confidence level (see Table I). This result suggests $N_{eff} = 3$ is inconsistent with the data with $\sim 95\%$ confidence.

We then allow the curvature to vary, to determine how assuming flatness affects the constraints on N_{eff}. Fig. 2 demonstrates the correlation between Ω_k and N_{eff}, which agrees with the prediction from Section II. Interestingly, the effect of the additional CMB datasets (ACT and SPT) increases the correlation between these parameters with respect to WMAP 7-year data alone. This may be due in part to the considerable improvement in N_{eff} whereas the uncertainty in the curvature is not noticeably improved by the addition of small scale anisotropy
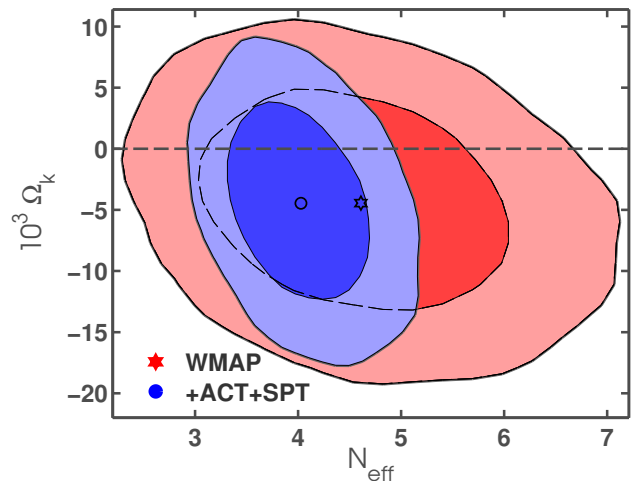

FIG. 2. Correlation between Ω_k and N_{eff}. The credible intervals are given at the 68% and 95% confidence levels and the markers indicate the locations of the marginalized values. WMAP+BAO+H_0 is shown in red while WMAP+ACT+SPT+BAO+H_0 is in blue. Note that the effect of adding additional datasets is significant.

measurements. These results suggest that an open universe with fewer neutrinos would look similar to a flat universe with more neutrinos. We also note that when including N_{eff} as a free parameter in the ΛCDM+Ω_k model, the 1σ constraint of $\Omega_k = -0.0023^{+0.0054}_{-0.0056}$ found in Ref. [3] does not deteriorate significantly for the same combination of datasets (i.e. WMAP+BAO+H_0). This is due to the presence of the BAO data and the H_0 prior in the analysis, since both probes are sensitive to the geometry of the Universe. Therefore, BAO and H_0 help to break the degeneracy between N_{eff} and Ω_k.

Table II provides a summary of parameter values for runs where Ω_k and N_{eff} vary. Here we find $N_{eff} = 4.03 \pm 0.45$ and $10^3\Omega_k = -4.46 \pm 5.24$ at the 68% confidence level. Therefore, even when Ω_k is allowed to vary, $N_{eff} = 3$ is still disfavored with $\sim 95\%$ confidence. Note that the increased value for H_0 is indicative of the known correlation between H_0 and Ω_k. We provide an equiva-

Parameter	WMAP7+N_{eff}+Ω_k	...+ACT+SPT
$100\Omega_b h^2$	2.26 ± 0.056	2.27 ± 0.045
$\Omega_c h^2$	0.136 ± 0.0169	0.129 ± 0.00915
Ω_Λ	0.721 ± 0.0179	0.723 ± 0.0158
n_s	0.9837 ± 0.0157	0.9863 ± 0.0147
τ	0.0887 ± 0.0148	0.0894 ± 0.0149
H_0 (km/s/Mpc)	74.88 ± 3.40	73.44 ± 2.03
N_{eff}	4.61 ± 0.96	4.03 ± 0.45
$10^3\,\Omega_k$	-4.45 ± 5.85	-4.46 ± 5.24

TABLE II. Summary of constraints while varying Ω_k and N_{eff}. All datasets include BAO and H_0 for improved parameter constraints. Errors are at the 68% CL. See Fig. 2.

Parameter	WMAP7+BAO+H_0	...+ACT+SPT
$100\Omega_b h^2$	2.26 ± 0.053	2.23 ± 0.038
$\Omega_c h^2$	0.112 ± 0.0036	0.111 ± 0.0029
Ω_Λ	0.719 ± 0.0182	0.726 ± 0.0154
n_s	0.968 ± 0.0124	0.963 ± 0.0092
τ	0.0897 ± 0.015	0.0873 ± 0.014
H_0 (km/s/Mpc)	69.2 ± 1.6	69.9 ± 1.37
$\sum m_\nu$	< 0.57 eV	< 0.45 eV

TABLE III. Summary of the constraint on the sum of the neutrino masses. All datasets include BAO and H_0 for improved parameter constraints. Errors are at the 68% CL except for $\sum m_\nu$, which is quoted as a 95% upper limit.

lent marginalized contour plot of H_0 vs. N_{eff} to emphasize the connection (see Fig. 3).

We now turn to the question of how well the datasets are able to constrain $\sum m_\nu$. Table III shows the results from WMAP in the first column and the result of adding the additional datasets in the final column. Although the constraint greatly improves the two sigma limit for the masses, this is not enough to favor either the standard or inverted hierarchy. However, this is not a surprise because none of the datasets are sensitive enough on their own. Forthcoming data from the Planck experiment and other future experiments will likely improve the mass constraint [37].

Finally, we investigate the effect of assuming flatness while determining an upper bound on $\sum m_\nu$. We investigate two models. The first assumes three degenerate massive neutrinos, while the second allows for additional relativistic species accounted by $\Delta N_{eff} > 0$. We define the correlation coefficient ρ_{ij} as the ratio of the off-diagonal term of the covariance matrix σ_{ij} to the 1σ errors $\sigma_i \sigma_j$, so that for two parameters denoted by i and j we have $\rho_{ij} = \sigma_{ij}/\sigma_i \sigma_j$. Figure 4 shows that $\sum m_\nu$ and Ω_k are strongly correlated with a correlation coefficient of $\rho_{\Omega_k \sum m_\nu} = 0.78$ for both models ($\Delta N_{eff} = 0$ and $\Delta N_{eff} > 0$). Furthermore, the degeneracy considerably increases the uncertainty in the sum of the neutrino masses. In fact, with $\Omega_k \neq 0$ the 95% upper limit on
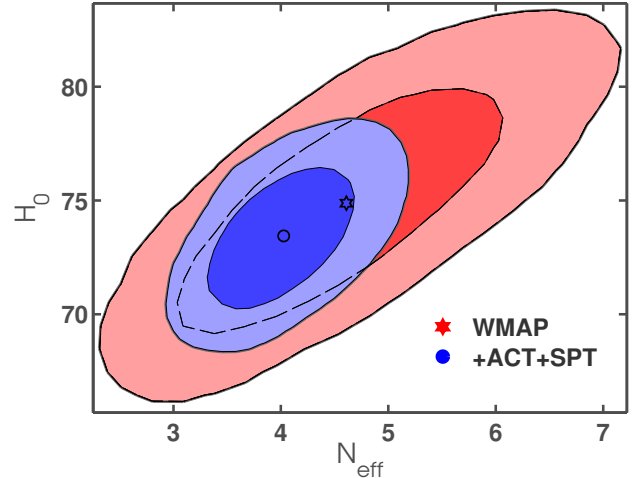

FIG. 3. Correlation between H_0 and N_{eff}. The credible intervals are given at the 68% and 95% confidence levels and the markers indicate the locations of the marginalized values. WMAP+BAO+H_0 is shown in red while WMAP+ACT+SPT+BAO+H_0 is in blue. In this case the effect of adding additional datasets is also significant.

$\sum m_\nu$ more than doubles with respect to the flat case: with $\sum m_\nu < 0.95$ eV for the model assuming only three massive neutrinos and $\sum m_\nu < 1.19$ eV for $\Delta N_{eff} > 0$. The strong correlation between curvature and mass is expected because massive neutrinos with $m_\nu < 0.3$ eV are still relativistic until recombination so they act as an additional radiative component. As a consequence the presence of such massive neutrinos shifts the time of matter-radiation equality a_{eq}. Recall the discussion in Section II where in this case lower mass neutrinos roughly correspond to higher N_{eff}. Neutrinos also leave an imprint on the CMB through the early Integrated Sachs-Wolfe effect (c.f. Refs. [38, 39]) which changes the position of acoustic peaks. This effect can be compensated for by a change in the geometry of the Universe, which weakens the constraints on both $\sum m_\nu$ and Ω_k. See Table IV for a summary of cosmological parameters when curvature and massive neutrinos are considered.

V. CONCLUSION

The resolution of the high effective neutrino number in cosmology remains an open question. However, additional neutrinos may be due to parameter degeneracy or other issues in statistical analysis rather than new physics. The focus of this paper has been an argument for correlation between the number of effective neutrinos N_{eff} and the curvature of the Universe Ω_k, which arises from the effect of these parameters on distance measurements. The qualitative argument is confirmed by a statistical analysis of CMB anisotropy measurements using CosmoMC.

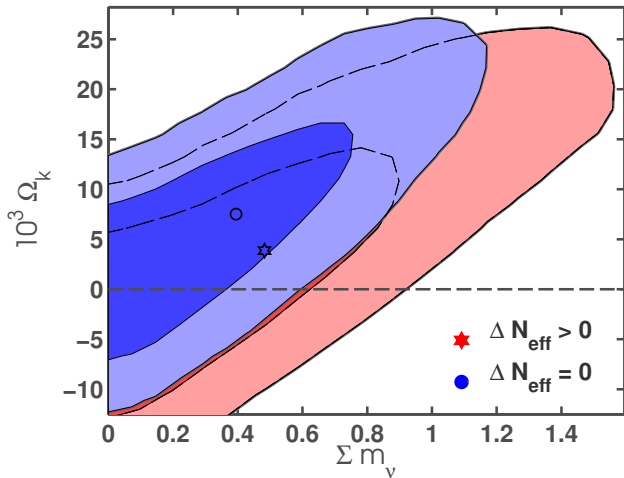In this paper we have shown that there is a correlation

between N_{eff} and Ω_k that gets stronger when SPT and ACT datasets are added to WMAP alone. However, even when Ω_k is allowed to vary, $N_{eff} = 3$ is still disfavored by the data with 95% confidence. Although the correlation favors a closed universe with $\Omega_k < 0$, if CMB data were to favor open models then the neutrino number would decrease as predicted. Perhaps the same element of the data that favors a closed universe may also be responsible for the trend toward a higher N_{eff}. More importantly, we find a strong correlation between curvature and the sum of the neutrino masses.

Future experiments will provide further insight into both N_{eff} and $\sum m_\nu$ [40]. Our results are consistent with the current understanding of the data available. The strongest constraints on these parameters from the statistical analysis assuming a flat universe are $N_{eff} = 3.89 \pm 0.41$ and $\sum m_\nu < 0.45$ eV with 95% confidence level using WMAP7+ACT+SPT+BAO+H_0. The constraints are weakened by degeneracy with the curvature parameter Ω_k. However, this still represents the continued effort toward significant improvements on parameter constraints in cosmology. Although the sum of the neutrino masses is significantly improved from the WMAP 7-year result of $\sum m_\nu < 0.57$ eV, the constraint is far from being sensitive enough to rule out one of the mass hierarchies. Furthermore, we have shown that the mass uncertainty more than doubles when $\Omega_k \neq 0$. Based on our results and the estimated quality of data for Planck and other experiments, it should be possible to determine the existence or nonexistence of sterile radiation to much greater confidence in the near future.

FIG. 4. Comparison of the correlation between Ω_k and $\sum m_\nu$ under the two ΔN_{eff} models. The model with three massive neutrinos is shown in blue while the model with additional relativistic species is in red. Intervals are given at the 68% and 95% confidence levels and markers indicate the locations of the marginalized values. Datasets include WMAP7+ACT+SPT+BAO+H_0. The addition of curvature allows $\sum m_\nu$ to be more than twice the previous constraint.

Parameter	$\Delta N_{eff} = 0$	$\Delta N_{eff} > 0$
$100\Omega_b h^2$	2.24 ± 0.043	2.26 ± 0.049
$\Omega_c h^2$	0.118 ± 0.0063	0.134 ± 0.0105
Ω_Λ	0.711 ± 0.0216	0.703 ± 0.0239
n_s	0.967 ± 0.011	0.982 ± 0.015
τ	0.0864 ± 0.0144	0.0890 ± 0.0145
H_0 (km/s/Mpc)	70.6 ± 1.62	73.1 ± 2.03
$10^3\,\Omega_k$	7.52 ± 7.74	3.46 ± 8.69
$\sum m_\nu$	< 0.95 eV	< 1.19 eV
ΔN_{eff}	0	0.995 ± 0.430

TABLE IV. Summary of the constraint on the sum of the neutrino masses when $\Omega_k \neq 0$. ΔN_{eff} is an additional relativistic contribution after considering 3.046 massive neutrinos. Datasets include WMAP7+ACT+SPT+BAO+H_0. Errors are at the 68% CL except for $\sum m_\nu$, which is quoted as a 95% upper limit.

VI. ACKNOWLEDGMENTS

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